Splitting of Andreev levels in a Josephson junction by spin-orbit coupling

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We consider the effect of spin-orbit coupling on the energy levels of a single-channel Josephson junction below the superconducting gap. We investigate quantitatively the level splitting arising from the combined effect of spin-orbit coupling and the time-reversal symmetry breaking by the phase difference between the superconductors. Using the scattering matrix approach, we establish a simple connection between the quantum mechanical time delay matrix and the effective Hamiltonian for the level splitting. As an application, we calculate the distribution of level splittings for an ensemble of chaotic Josephson junctions. The distribution falls off as a power law for large splittings, unlike the exponentially decaying splitting distribution given by the Wigner surmise—which applies for normal chaotic quantum dots with spin-orbit coupling in the case that the time-reversal symmetry breaking is due to a magnetic field.

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I. INTRODUCTION

A Josephson junction is a weak link between two superconductors with an adjustable phase difference ϕ. The weak link may be a tunnel barrier or a normal metal. Figure 1 shows, for example, a Josephson junction consisting of a small piece of normal metal (a quantum dot), connected to the superconductors by a pair of narrow constrictions (quantum point contacts). The excitation spectrum below the superconducting gap Δ consists of discrete energies, called Andreev levels. In zero magnetic field, the energy levels ε_n are determined by the normal-state transmission eigenvalues T_n if $\Delta \ll \hbar/\tau_{\mathrm{dw}}$, where τ_{dw} is the dwell time of an electron in the normal region (before it is converted into a hole by Andreev reflection at the superconductor). The relationship is[1]

$$\varepsilon_n = \Delta\sqrt{1 - T_n \sin^2(\phi/2)} + \mathcal{O}(\Delta^2 \tau_{\mathrm{dw}}/\hbar). \tag{1}$$

Each level is twofold spin degenerate (Andreev doublet).

Recently, the effect of spin-orbit coupling on Josephson junctions became a subject of investigation.[2–6] This is a subtle effect for the following reason: On the one hand, in the absence of magnetic fields, the normal-state transmission eigenvalues T_n are Kramers degenerate because of the time-reversal invariance of the normal system. On the other hand, one would expect a breaking of the degeneracy of the Andreev doublets because the phase difference between the superconducting contacts breaks the time-reversal symmetry of the system. Still, to leading order in $\Delta\tau_{\mathrm{dw}}/\hbar$, the one-to-one relationship [Eq. (1)] between ε_n and T_n ensures that the Andreev levels remain degenerate for nonzero ϕ. As was pointed out by Chtchelkatchev and Nazarov,[4] to see a splitting of the Andreev doublets as a result of the combined effect of spin-rotation symmetry breaking by spin-orbit coupling and time-reversal symmetry breaking by the phase difference, one has to go beyond the leading order in $\Delta\tau_{\mathrm{dw}}/\hbar$. This tunable level splitting was exploited in a proposal of Andreev qubits for quantum computation.[4]

In this work, we examine the splitting of the Andreev doublets quantitatively by calculating the first order correction to the energy levels in the small parameter $\Delta\tau_{\mathrm{dw}}/\hbar$. We concentrate our attention on the case when the quantum point contacts support one propagating mode each. We give a simple relation between the effective Hamiltonian for the level splitting of Chtchelkatchev and Nazarov[4] and the Wigner-Smith time delay matrix,

$$Q = -iS^{\dagger}\frac{dS}{d\varepsilon}, \tag{2}$$

where S is the scattering matrix of the normal system. As an application, we calculate how the splittings are distributed for an ensemble of systems where the two superconductors are connected by a chaotic quantum dot, assuming that the spin-orbit coupling in the dot is strong enough that the dot Hamiltonian can be modeled as a member of the symplectic ensemble of random matrix theory (RMT).[7,8] The present study in the regime $\Delta \ll \hbar/\tau_{\mathrm{dw}}$ complements earlier work[9,10] in the opposite regime $\Delta \gg \hbar/\tau_{\mathrm{dw}}$. (The systems studied in our paper correspond to symmetry class D in the classification of Ref. 9.)

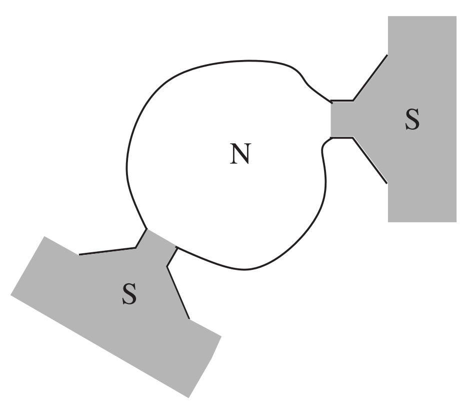

FIG. 1. Sketch of a quantum dot Josephson junction: the quantum dot (N) is connected to two superconductors (S) by point contacts. Spin-orbit coupling splits the energy levels of the system when the superconductors have a nonzero phase difference.

The paper is organized as follows. In Sec. II, we employ the scattering matrix approach for calculating the first order correction in $\Delta\tau_{dw}/\hbar$ to the Andreev levels and obtain the effective Hamiltonian for the level splitting in terms of the time delay matrix Q. For simplicity, we consider the single-channel case in Sec. II and give the multichannel extension in Appendix A. We apply our single-channel formula to a calculation of the splitting distribution for an ensemble of chaotic Josephson junctions in Sec. III. We conclude in Sec. IV with a comparison of the splitting distribution of the Andreev doublets and the Wigner surmise of RMT.

II. SPLITTING HAMILTONIAN AND WIGNER-SMITH MATRIX

For energies below the superconducting gap Δ, the Josephson junction supports bound states, with excitation energies given by the roots of the secular equation,[1]

$$\mathrm{Det}[\mathbb{1} - \alpha(\varepsilon)^2 r_A^* S_e(\varepsilon) r_A S_h(\varepsilon)] = 0, \tag{3}$$

where

$$\alpha = \exp\left[-i \arccos\left(\frac{\varepsilon}{\Delta}\right)\right], \quad r_A = \begin{pmatrix} e^{i\phi/2}\mathbb{1} & 0 \\ 0 & e^{-i\phi/2}\mathbb{1} \end{pmatrix}. \tag{4}$$

The matrices $S_e(\varepsilon)$ and $S_h(\varepsilon)$ are the scattering matrices of the normal system for electrons and holes. They are related as

$$S_h(\varepsilon) = \mathcal{T} S_e(-\varepsilon)\mathcal{T}^{-1}, \tag{5}$$

where $\mathcal{T}=i\sigma_2 K$ is the time-reversal operator for spin-1/2 particles. The matrix σ_2 is the second Pauli matrix acting on the spin degree of freedom and K is the operator of complex conjugation. Relation (5) reflects the fact that in the normal part the dynamics of the holes is governed by the Hamiltonian[11]

$$H_h = -\mathcal{T} H_e \mathcal{T}^{-1}, \tag{6}$$

the negative of the time reversed electron Hamiltonian H_e.

The matrix r_A describes the conversion of electrons into holes by Andreev reflection at the interfaces with the superconductors. The phase shift $\alpha(\varepsilon)$ is acquired upon Andreev reflection because of the penetration of the wave function into the superconductor.

We consider the case when the normal part is time-reversal invariant, which imposes the self duality condition $S=\sigma_2 S^T \sigma_2$ on the scattering matrix. (The superscript T refers to matrix transposition.) For $\phi=0$, the solutions of Eq. (3) have a twofold degeneracy known as Kramers degeneracy. (Kramers degeneracy is a generalization of spin degeneracy to cases when spin-rotation symmetry is broken but time-reversal symmetry is preserved.)

The typical elements of $S_e(\varepsilon)$ change significantly if ε is changed on the scale of $\hbar/\tau_{dw}$; therefore, to leading order in $\Delta\tau_{dw}/\hbar$, one can neglect the energy dependence of $S_e(\varepsilon)$, and take it at the Fermi energy, $S_e(\varepsilon)\approx S_e(0)$. Making use of the self-duality of the scattering matrix and introducing the usual block structure,

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \tag{7}$$

the secular equation [Eq. (3)] can be simplified to[1]

$$\mathrm{Det}\left[\left(1-\frac{\varepsilon^2}{\Delta^2}\right) - t^\dagger t \sin^2\left(\frac{\phi}{2}\right)\right] = 0. \tag{8}$$

From this equation follows relation (1) between the energies and the transmission eigenvalues.

The correction of order $\Delta^2\tau_{dw}/\hbar$ comes from considering the energy dependence of the scattering matrix to first order, $S(\varepsilon)\approx S(0)+(dS/d\varepsilon)\varepsilon$. For simplicity, we restrict ourselves here to the case of two single-channel point contacts. (The extension to multichannel point contacts is given in Appendix A.) For single-channel point contacts, the self-duality of the scattering matrix implies

$$r = \rho\mathbb{1}_2, \quad r' = \rho'\mathbb{1}_2, \quad t' = \sigma_2 t^T \sigma_2, \quad t = \sqrt{T}U, \tag{9}$$

where ρ and ρ' are complex numbers, $\mathbb{1}_2$ is the 2×2 unit matrix, $1\geq T\geq 0$, and U is a 2×2 unitary matrix. Writing the energy as $\varepsilon_0+\delta\varepsilon$ with

$$\varepsilon_0 = \Delta\sqrt{1-T\sin^2(\phi/2)}, \tag{10}$$

and keeping terms up to linear order in the small quantities $\delta\varepsilon=\mathcal{O}(\Delta^2\tau_{dw}/\hbar)$ and $\Delta\tau_{dw}/\hbar$, one finds the eigenvalue equation,

$$\mathrm{Det}\left[\frac{\Delta^2}{4}(\sigma_2 Q_{11}^T\sigma_2 - Q_{11})\sin(\phi) - \frac{\Delta^2}{4}(\mathrm{Tr}\, Q)\frac{\varepsilon_0}{\Delta}\sqrt{1-\frac{\varepsilon_0^2}{\Delta^2}}\mathbb{1}_2 - \delta\varepsilon\right] = 0 \tag{11}$$

for the energy correction $\delta\varepsilon$. The matrix Q has the block structure,

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}, \tag{12}$$

inherited from the transmission-reflection block structure [Eq. (7)] of the scattering matrix.

The second term in the determinant [Eq. (11)] shifts both eigenvalues by the same amount $\delta\varepsilon_{shift}$, while the first, manifestly traceless term is responsible for the splitting $\pm\delta\varepsilon_{split}$ of the doublet. We see that the splitting is determined by the effective Hamiltonian

$$H_{eff} = \Delta\frac{\tau_{dw}\Delta}{\hbar}\Sigma\sin(\phi), \tag{13}$$

with Σ a traceless Hermitian 2×2 matrix having matrix elements of order unity. This is the result of Chtchelkatchev and Nazarov.[4] Our analysis gives an explicit relation[16] between the matrix Σ and the time delay matrix Q,

$$\Sigma = \frac{\hbar}{4\tau_{dw}}(\sigma_2 Q_{11}^T \sigma_2 - Q_{11}). \tag{14}$$

This is the key relation that will allow us, in the next section, to calculate the level splitting distribution from the known properties of the time delay matrix in a chaotic system.

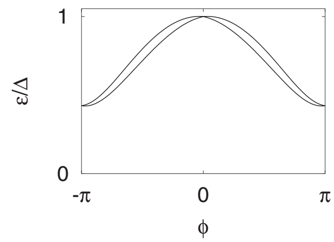

FIG. 2. A schematic illustration of the splitting of the Andreev doublet as a function of the phase difference ϕ for a single-channel Josephson junction with spin-orbit coupling. The energies are the sum of a degenerate part $\varepsilon_0 + \delta\varepsilon_{\text{shift}}$ that is even in ϕ and a splitting $\pm\delta\varepsilon_{\text{split}}$ that is odd in ϕ, as explained in the text. The maximal splitting is reached at $\phi = \pi/2$.

We conclude this section with a symmetry consideration. The shift $\delta\varepsilon_{\text{shift}}$ is even in ϕ, just like the zeroth order term ε_0. In contrast, the splitting $\delta\varepsilon_{\text{split}}$ is odd in ϕ. This is in accord with the symmetry of the Hamiltonian H that gives the full excitation spectrum of the Josephson junction. Under time reversal, in our case of a time-reversal invariant normal part, it transforms as $\mathcal{T}H(\phi)\mathcal{T}^{-1} = H(-\phi)$; therefore, for an eigenstate Ψ, one has

$$H(\phi)\Psi(\phi) = \varepsilon(\phi)\Psi(\phi),$$

$$H(\phi)\mathcal{T}\Psi(-\phi) = \varepsilon(-\phi)\mathcal{T}\Psi(-\phi). \tag{15}$$

An Andreev doublet is therefore of the form $\{\varepsilon(\phi), \varepsilon(-\phi)\}$. The decomposition of $\varepsilon(\phi)$ into even and odd parts in ϕ amounts to a decomposition of the doublet into a degenerate even part and an odd splitting part. The resulting ϕ dependence of the doublet is shown schematically in Fig. 2.

III. SPLITTING DISTRIBUTION IN CHAOTIC JOSEPHSON JUNCTIONS

As an application of our general result [Eq. (14)], we calculate how the level splittings are distributed for an ensemble of Josephson junctions where the normal part is a chaotic quantum dot. We assume that the spin-orbit coupling inside the dot is strong enough that the dot Hamiltonian can be modeled as a member of the symplectic ensemble of RMT, i.e., that the spin-orbit time τ_{so} is much shorter than τ_{dw}. In this limit, the splitting distribution becomes independent of τ_{so}. (In the opposite limit $\tau_{\text{so}} \gg \tau_{\text{dw}}$, the splitting increases linearly with $1/\tau_{\text{so}}$.)

The splitting distribution can be obtained from the known distribution of the scattering matrix,[7] and of the dimensionless symmetrized Wigner-Smith matrix,[12]

$$Q_E = -i\frac{\hbar}{\tau_{\text{dw}}} S^{-1/2}(dS/d\varepsilon)S^{-1/2}. \tag{16}$$

The distributions of S and Q_E are independent,[12] which makes it advantageous to express Q in terms of S and Q_E,

$$Q = \frac{\tau_{\text{dw}}}{\hbar} S^{-1/2} Q_E S^{1/2}. \tag{17}$$

In the single-channel case, one has

$$Q_E = M_1 \begin{pmatrix} 1/\gamma_1 \mathbb{1}_2 & 0 \\ 0 & 1/\gamma_2 \mathbb{1}_2 \end{pmatrix} M_1^\dagger,$$

$$S = M_2 \begin{pmatrix} e^{i\varphi_1}\mathbb{1}_2 & 0 \\ 0 & e^{i\varphi_2}\mathbb{1}_2 \end{pmatrix} M_2^\dagger. \tag{18}$$

The rates γ_n are distributed according to[12]

$$P(\gamma_1, \gamma_2) \propto |\gamma_1 - \gamma_2|^4 \gamma_1^4 \gamma_2^4 \exp[-4(\gamma_1 + \gamma_2)]. \tag{19}$$

The distribution of the phases ϕ_n is[7]

$$P(\phi_1, \phi_2) \propto |e^{i\phi_1} - e^{i\phi_2}|^4. \tag{20}$$

The matrices of eigenvectors M_1 and M_2 are members of the group Sp(2) of 4×4 unitary symplectic matrices and are uniformly distributed with respect to the Haar measure of the group.[7,12] The Haar measure is given as

$$d\mu \propto \sqrt{|\text{Det } g|}\,\Pi_j dx_j, \tag{21}$$

in terms of the metric tensor g, defined by

$$\text{Tr}(dM dM^\dagger) = \sum_{ij} g_{ij} dx_i dx_j. \tag{22}$$

Here, $\{x_i\}$ is a set of independent variables parameterizing the Sp(2) matrix M.

A convenient choice to parametrize Sp(2) is the decomposition

$$M = \begin{pmatrix} \cos(\theta) & \sin(\theta)W \\ -\sin(\theta)W & \cos(\theta) \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}, \tag{23}$$

where W, U, and V are SU(2) matrices, and $\theta \in [0, \pi/2]$. It is seen that the SU(2)$\otimes$SU(2) factor corresponding to the block-diagonal matrix with U and V cancels from the spectral decomposition [Eq. (18)] of Q_E and S. Using the Euler angle parametrization for SU(2),

$$U = \begin{pmatrix} e^{-i(\phi_U+\psi_U)/2}\cos(\theta_U/2) & -e^{i(\psi_U-\phi_U)/2}\sin(\theta_U/2) \\ e^{i(\phi_U-\psi_U)/2}\sin(\theta_U/2) & e^{i(\phi_U+\psi_U)/2}\cos(\theta_U/2) \end{pmatrix},$$

$$\phi_U \in [0, 2\pi], \quad \psi_U \in [0, 4\pi], \quad \theta_U \in [0, \pi], \tag{24}$$

and similarly for matrices V and W, one finds that the Haar measure on Sp(2) corresponding to the chosen parametrization is

$$d\mu(M) \propto \sin^3(\theta)\cos^3(\theta)d\theta \prod_{j=U,V,W} \sin(\theta_j)d\phi_j d\theta_j d\psi_j. \tag{25}$$

We define the maximal dimensionless splitting q of the Andreev levels (reached at $\phi = \pi/2$) by the formula

$$\delta\varepsilon_{\text{split}} = q\Delta\frac{\Delta\tau_{\text{dw}}}{\hbar}\sin(\phi). \tag{26}$$

The distribution of q is given by

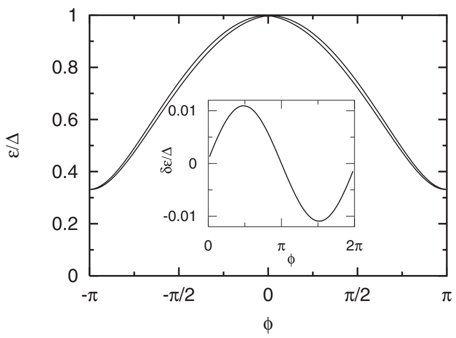

FIG. 3. The evolution of the Andreev levels of a single sample of the spin kicked rotator as a function of the superconducting phase difference ϕ. The inset shows the sinusoidal dependence of the level splitting on ϕ.

$$P(q) = \int d\mu(S)d\mu(Q_E)\delta[q - \sqrt{-\mathrm{Det}(\Sigma)}],$$

$$d\mu(Q_E) = d\mu(M_1)d\gamma_1 d\gamma_2 P(\gamma_1,\gamma_2),$$

$$d\mu(S) = d\mu(M_2)d\varphi_1 d\varphi_2 P(\varphi_1,\varphi_2). \tag{27}$$

Equation (27) can be evaluated numerically. The resulting distribution is shown in Fig. 4. The first two moments of q are

$$\langle q\rangle = 0.181, \quad \sqrt{\langle q^2\rangle - \langle q\rangle^2} = 0.152. \tag{28}$$

The analysis of the asymptotic behavior of the integral in Eq. (27) shows that near zero the splitting distribution behaves as

$$P(q) \sim q^2 \quad (q \to 0). \tag{29}$$

For large splittings, we find

$$P(q) \sim q^{-6} \quad (q \to \infty). \tag{30}$$

In order to check our prediction [Eq. (27)] for the level splitting distribution, we have numerically simulated the chaotic quantum dot Josephson junction of Fig. 1 using the spin kicked rotator.[13,14] The spin kicked rotator is a dynamical model, from which one can extract scattering matrices characteristic of chaotic cavities. These scattering matrices are given by

$$S(\varepsilon) = \mathcal{P}[e^{-i\varepsilon} - \mathcal{F}(1-\mathcal{P}^T\mathcal{P})]^{-1}\mathcal{F}\mathcal{P}^T, \tag{31}$$

where $\mathcal{F}$ is a $2M\times 2M$ matrix giving the stroboscopic time evolution of the model and $\mathcal{P}$ is a $4\times 2M$ projection matrix projecting onto the two single-channel point contacts (the factors of 2 in the dimensions are because of the spin). The quasienergy ε plays the role of the energy variable, measured in units of $\hbar/t_0$ with t_0 the stroboscopic time. For a more detailed description of this numerical model, we refer the reader to Ref. 14.

Scattering matrices generated through Eq. (31) are inserted into the secular Eq. (3), and the roots are found by varying the quasienergy. The dwell time in this model is $\tau_{\mathrm{dw}}=M/2$ (again in units of t_0). We take $M=100$ and $\Delta=2\times 10^{-4}$ (in units of $\hbar/t_0$), so that $\Delta\tau_{\mathrm{dw}}/\hbar=10^{-2}\ll 1$.

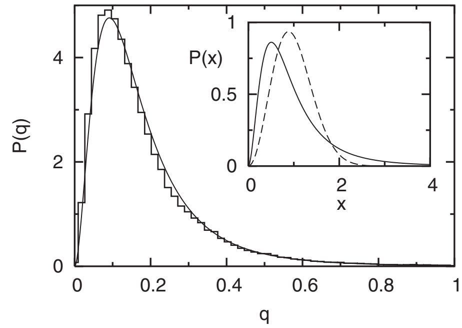

FIG. 4. Main plot: distribution of the maximal splitting of the Andreev levels (reached at $\phi=\pi/2$) in units of $\Delta^2\tau_{\mathrm{dw}}/\hbar$. The smooth curve is the prediction of random matrix theory calculated from Eq. (27); the histogram is the result of a numerical simulation using the spin kicked rotator. Inset: comparison of the Andreev doublet splitting distribution (solid line) and the Wigner surmise (dashed line). For this comparison, the energies are rescaled such that the mean of the distributions is unity.

Throughout the simulations, the strength of spin-orbit coupling was characterized by $\tau_{\mathrm{dw}}/\tau_{\mathrm{so}}=625$. In Fig. 3, we show the dependence of a single Andreev doublet in the dynamical model on the superconducting phase difference ϕ. The level splitting is sinusoidal as predicted. By sampling about 10^5 different $\mathcal{F}$, P, and ϕ, we numerically obtain the distribution $P(q)$ shown in Fig. 4 together with the analytical result Eq. (27). The first two moments of the distribution obtained from our simulation are

$$\langle q\rangle = 0.181, \quad \sqrt{\langle q^2\rangle - \langle q\rangle^2} = 0.160, \tag{32}$$

in a close agreement with the analytical predictions [Eq. (28)].

IV. DISCUSSION

A. Summary

We have investigated the effect of spin-orbit coupling on the subgap spectrum of single-channel Josephson junctions. Using the scattering matrix approach and considering the energy dependence of the scattering matrix to first order, we obtained a simple relation [Eq. (14)] between the effective Hamiltonian governing the level splitting and the quantum mechanical time delay matrix $Q=-iS^\dagger dS/d\varepsilon$. This relation allowed us to find the splitting distribution for an ensemble of chaotic Josephson junctions using the known properties of Q. We verified our result numerically by simulating the chaotic Josephson junction using the spin kicked rotator, and we found excellent agreement.

Experimentally, our results are relevant for quantum dots with ballistic point contacts, such as studied in Ref. 17. Effects of the Coulomb blockade, which dominate when the point contacts contain a tunnel barrier, should be relatively unimportant in this case. The observation of weak antilocalization, as reported in Ref. 17, is a signature of the regime $\tau_{\mathrm{so}}\ll\tau_{\mathrm{dw}}$ considered in Sec. III. The splitting of the Andreev levels can be seen in the Josephson current if a nonequilibrium population of the levels is induced. (There is no effect in equilibrium.) Lundin *et al.*[18] have proposed a microwave irradiation spectroscopy method that might be used to measure the splitting.

B. Comparison of the splitting distribution with the Wigner surmise

In the inset of Fig. 4, we compare the splitting distribution of the Andreev doublet with the Wigner surmise of RMT,[8]

$$P_W(x) = \frac{32}{\pi^2} x^2 \exp\left(-\frac{4x^2}{\pi}\right). \tag{33}$$

(For this comparison, the energy scale is set such that the average splitting is unity.) The motivation behind this comparison is the fact that the Wigner surmise is also a splitting distribution: as shown in Appendix B, it describes the distribution of the splittings of Kramers doublets for normal chaotic quantum dots with spin-orbit coupling in the case that the time-reversal symmetry is broken by a magnetic field.

At small splittings, both P and P_W decay quadratically. This quadratic decay is a generic feature of the splitting of a Kramers degenerate level due to time-reversal symmetry breaking. It follows from the fact that the splitting Hamiltonian is a 2×2 Hermitian traceless matrix without further symmetries and from a power counting argument[15] similar to the one leading to the quadratic decay of P_W.

While at small splittings the two distributions decay in the same way, we find qualitative differences in the opposite limit. At large splittings, P decays like a power law in contrast to the exponential decay of P_W [cf. Eqs. (30) and (33)].

We attribute the deviation of P from the Wigner surmise to the nonuniform way in which time-reversal symmetry is broken: While the magnetic field in Appendix B acts *uniformly* throughout the normal quantum dot, the superconducting phase difference in the Josephson junction acts *nonuniformly* at the point contacts. The nonuniformity is obvious in the dynamical model, where the point contacts are introduced at a specific position in phase space. In the RMT description, there is no notion of a phase space, but still the coupling to the point contacts is introduced in a nonuniform way by coupling to a small subset of matrix elements of the random Hamiltonian. (This can be seen from the parametrization of the scattering matrix in the Hamiltonian approach.[7])

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APPENDIX A: SPLITTING HAMILTONIAN FOR MULTICHANNEL JOSEPHSON JUNCTIONS

We generalize relation (14) between the splitting Hamiltonian and the time delay matrix to the case that each of the two point contacts supports $N/2$ propagating modes. (The single-channel case of Sec. II therefore corresponds to $N=2$.) In the multichannel case, after the steps leading to Eq. (11), one arrives at the equation

$$\mathrm{Det}\left[H_0 + \frac{\Delta^2}{2} K - \delta\varepsilon\right] = 0, \tag{A1}$$

where

$$H_0 = \frac{\Delta^2}{2\varepsilon_n^{(0)}}\left[1 - \left(\frac{\varepsilon_n^{(0)}}{\Delta}\right)^2 - t^\dagger t \sin^2\left(\frac{\phi}{2}\right)\right], \tag{A2}$$

$$\varepsilon_n^{(0)} = \Delta\sqrt{1 - T_n \sin^2(\phi/2)}, \tag{A3}$$

and K is a matrix with elements of order $\tau_{\mathrm{dw}}/\hbar$. An eigenvector of $t^\dagger t$ with eigenvalue T_n is also an eigenvector of H_0 with zero eigenvalue. The first order correction to the zeroth order energy $\varepsilon_n^{(0)}$ is the first order perturbative correction to this zero eigenvalue.

We introduce the $N\times 2$ matrices W_n and W_n' which contain the two orthonormal eigenvectors of, respectively, $t^\dagger t$ and $t'^\dagger t'$, both corresponding to the eigenvalue T_n. In terms of these matrices, we define the matrices q_{1n} and q_{2n} by

$$q_{1n} = W_n^\dagger Q_{11} W_n, \quad q_{2n} = W_n'^\dagger Q_{22} W_n'. \tag{A4}$$

We find that the shift of the Andreev doublet at $\varepsilon_n^{(0)}$ is given by

$$\delta\varepsilon_n^{\mathrm{shift}} = -\frac{\Delta^2}{4}\frac{\varepsilon_n^{(0)}}{\Delta}\sqrt{1 - (\varepsilon_n^{(0)}/\Delta)^2}(\mathrm{Tr}\, q_{1n} + \mathrm{Tr}\, q_{2n}), \tag{A5}$$

while the splitting $\delta\varepsilon_n^{\mathrm{split}}$ is given by the two eigenvalues of the traceless Hermitian matrix,

$$H_{\mathrm{eff}}^{(n)} = \frac{\Delta^2}{4}(\sigma_2 q_{1n}^T \sigma_2 - q_{1n})\sin(\phi). \tag{A6}$$

APPENDIX B: SPLITTING DISTRIBUTION FOR NORMAL CHAOTIC QUANTUM DOTS

We calculate the splitting distribution of a Kramers degenerate level for normal chaotic quantum dots with spin-orbit coupling, in the case that the time-reversal symmetry is broken by a magnetic field.

The Hamiltonian of the system is decomposed into two parts,

$$H = H_0 + A, \quad H_0^\dagger = H_0, \quad A^\dagger = A, \tag{B1}$$

where H_0 and A are $2M\times 2M$ matrices (the factor of 2 is due to the spin). They satisfy

$$\mathcal{T} H_0 \mathcal{T}^{-1} = H_0, \quad \mathcal{T} A \mathcal{T}^{-1} = -A. \tag{B2}$$

The matrix H_0 models the time-reversal invariant part of the Hamiltonian and A is a time-reversal symmetry breaking term.

The eigenvalues of H_0 are doubly degenerate (Kramers degeneracy). Considering a doublet with energy E_0, with corresponding eigenvectors u_1, $u_2=\mathcal{T}u_1$,

$$H_0 u_1 = E_0 u_1, \quad H_0 u_2 = E_0 u_2, \tag{B3}$$

and treating A as a perturbation, first order degenerate perturbation theory leads to the splitting of the Kramers doublet by an amount $\pm\delta\varepsilon_{\mathrm{split}}$. We find

$$\delta\varepsilon_{\mathrm{split}} = \sqrt{\langle u_1, A u_1\rangle^2 + |\langle u_1, A u_2\rangle|^2}. \tag{B4}$$

For chaotic billiards, the splitting distribution is given by[8]

$$P(\lambda) = \int dU \rho(U) \int dA P(A) \delta(\lambda - \delta\varepsilon_{\mathrm{split}}), \tag{B5}$$

where U is the matrix of eigenvectors of H_0, distributed according to $\rho(U)$. [The form of $\rho(U)$ is not needed for the derivation.] The matrix A has distribution

$$P(A) \propto \exp(-v^2 \operatorname{Tr} A^2), \tag{B6}$$

where v is a positive number. Using the fact that $P(A)dA$ is invariant under a unitary transformation with the matrix of eigenvectors of H_0, one finds

$$P(\lambda) = \int da db dc P(a,b,c) \delta(\lambda - \sqrt{a^2 + b^2 + c^2}), \tag{B7}$$

where

$$P(a,b,c) \propto \exp[-2v^2(a^2 + b^2 + c^2)]. \tag{B8}$$

After changing to polar coordinates, integral (B7) can be evaluated straightforwardly, and after rescaling from λ to x, defined by $\int dx P(x) x = 1$, one arrives at the Wigner surmise [Eq. (33)].

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